TI-83 Calculator Instructions for $\chi^2$-Tests (Independence & Goodness-of-Fit)

Chi square test for independence and Chi-square test for homogeneity of proportions: To see if two variables are independent of each other, perform the following steps:

- Type [MATRIX] [EDIT] [A] [ENTER]
- Then [m] [ENTER] [n] [ENTER], m being the number of rows in your data set (number of data values for variable 1) and n being the number of columns (number of data values for variable 2)
- Type in the observed data values into the $m \times n$ matrix [A].
- Select [QUIT]
- Select [STAT] [TESTS] [$\chi^2$-Test] [ENTER]
- For Observed: type [A] [ENTER]
- For Expected: type [B] [ENTER]
- Select [CALCULATE] [ENTER].

Performing a Chi-square Goodness of Fit Test: To see if a given data distribution fits a particular pattern of $p_1, p_2, \ldots, p_n$, perform the following steps and fool the calculator into "thinking" it is doing a $\chi^2$-test for independence. For those who are interested, see an explanation of why this works by clicking on the link immediately after the link to this page.

- Type [MATRIX] [EDIT] [A] [ENTER]
- Then type in [2] [ENTER] [n] [ENTER], n being the number of categories.
- Then fill in the entries of the matrix [A], the observed data in the first row and the "fake" observed data in the second row as below:

$$
\begin{bmatrix}
O_1 & O_2 & \ldots & O_n \\
p_1 \cdot 10^{20} & p_2 \cdot 10^{20} & \ldots & p_n \cdot 10^{20}
\end{bmatrix}
$$

- Then [QUIT]
- Select [STAT] [TESTS] [$\chi^2$-Test] [ENTER]
- For Observed: type [A] [ENTER]
- For Expected: type [B] [ENTER]
- Select [CALCULATE] [ENTER]
Why the TI-83 can be fooled into using the chi-square test (of independence) to correctly perform a chi-square goodness of fit test

This test uses data obtained from a random sample to check whether population data fit a particular distribution. The hypotheses are

\[ H_0 \quad : \quad \text{Data fit a certain distribution} \]
\[ H_1 \quad : \quad \text{Data do not fit this distribution} \]

Suppose the observed data/frequencies are given in first row of the following table, then if the expected distribution has the proportions \( p_1 : p_2 : \cdots : p_n \), the "fake" observed frequencies are obtained by multiplying a very large number, say \( B \), by each of the proportions as shown in the table:

| Categories | Category 1 | Category 2 | \( \cdots \) | Category \( n \) | Row Sums |
|------------|------------|------------|\| | \| | \| |
| Observed freq. | \( O_{11} \) | \( O_{12} \) | \( \cdots \) | \( O_{1n} \) | \( R_1 \) |
| Fake freq. | \( O_{21} = p_1B \) | \( O_{22} = p_2B \) | \( \cdots \) | \( O_{21} = p_nB \) | \( R_2 \) |
| Column Sums | \( C_1 \) | \( C_2 \) | \( \cdots \) | \( C_n \) | \( G \) |

**Table 1 : Observed Frequencies**

Here \( G \) represents the grand total of the actual and fake frequencies, which will be a very large number.

Note that The column sums are very large since they include the fake frequencies, and \( R_2 \) is very large since it is the sum of all the fake observed frequencies.

The expected frequencies are then computed as follows:

| Categories | Category 1 | Category 2 | \( \cdots \) | Category \( n \) |
|------------|------------|------------|\| | \| |
| Expected freq. | \( E_{11} = \frac{R_1 \cdot C_1}{G} \) | \( E_{12} = \frac{R_1 \cdot C_2}{G} \) | \( \cdots \) | \( E_{1n} = \frac{R_1 \cdot C_n}{G} \) |
| Fake freq. | \( E_{21} = \frac{R_1 O_{1j}}{R_1 + B} \) | \( E_{22} = \frac{R_1 O_{2j}}{R_1 + B} \) | \( \cdots \) | \( E_{2n} = \frac{R_1 O_{nj}}{R_1 + B} \) |

**Table 2 : Expected Frequencies**

Some algebra at this point will make things a little easier to handle later on:

\[ R_2 = B(p_1 + p_2 + \cdots + p_n) = B; \quad \text{since} \quad p_1 + p_2 + \cdots + p_n = 1, \]

so

\[ G = R_1 + R_2 = R_1 + B. \]

Then

\[ E_{1j} = \frac{R_1 \cdot C_j}{G}, \quad j = 1, 2, \ldots, n; \]

\[ = \frac{R_1 (O_{1j} + p_j B)}{R_1 + B}, \]

\[ = \frac{R_1 O_{1j} + p_j R_1 B}{R_1 + B}, \quad \text{or} \quad \frac{R_1}{R_1 + B} + \frac{p_j R_1}{R_1 + B} \]

\[ = \frac{R_1 O_{1j}}{R_1 + B} + \frac{p_j R_1}{R_1 + B} \cdot \frac{1}{1}, \]

and since \( B \) is taken as a very large number,

\[ \frac{R_1 O_{1j}}{R_1 + B} \approx 0 \quad \text{and} \quad \frac{R_1}{B} \approx 0 \]

so that

\[ E_{1j} \approx p_j R_1. \quad (1) \]
Similarly,
\[ E_{2j} = \frac{R_2 \cdot C_i}{G} = \frac{B(O_{1j} + p_jB)}{R_1 + B} = B\frac{O_{1j}}{R_1 + B} + B\frac{p_jB^2}{R_1 + B} = \frac{O_{1j}}{R_1/B + 1} + \frac{p_jB}{R_1/B + 1}, \]
and since \( B \) is taken as a very large number,
\[ R_1/B \approx 0 \]
so that
\[ E_{2j} \approx O_{1j} + p_jB. \]

Substituting the expressions from the approximations (1) and (2) into the expected frequencies table gives

<table>
<thead>
<tr>
<th>Categories</th>
<th>Category 1</th>
<th>Category 2</th>
<th>\cdots</th>
<th>Category n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. freq.</td>
<td>( E_{11} = p_1R_1 )</td>
<td>( E_{12} = p_2R_1 )</td>
<td>\cdots</td>
<td>( E_{1n} = p_nR_1 )</td>
</tr>
<tr>
<td>Fake freq.</td>
<td>( E_{21} = O_{11} + p_1B )</td>
<td>( E_{22} = O_{12} + p_2B )</td>
<td>\cdots</td>
<td>( E_{2n} = O_{1n} + p_nB )</td>
</tr>
</tbody>
</table>

Table 3: Expected Frequencies

The test value is obtained by computing the grand total of the entries in the following table

| \( (O_{11} - E_{11})^2 \) | \( (O_{12} - E_{12})^2 \) | \cdots | \( (O_{1n} - E_{1n})^2 \) |
| \( (p_1B - (O_{11} + p_1B))^2 \) | \( (p_2B - (O_{12} + p_2B))^2 \) | \cdots | \( (p_nB - (O_{1n} + p_nB))^2 \) |

\( \frac{O_{11}}{O_{11} + p_1B} \) | \( \frac{O_{12}}{O_{12} + p_2B} \) | \cdots | \( \frac{O_{1n}}{O_{1n} + p_nB} \)

The first row consists of "nice" numbers, i.e., no \( B \)'s. The second row can be simplified so that the table takes the form

| \( (O_{11} - E_{11})^2 \) | \( (O_{12} - E_{12})^2 \) | \cdots | \( (O_{1n} - E_{1n})^2 \) |
| \( E_{11} \) | \( E_{12} \) | \cdots | \( E_{1n} \) |

\[ \frac{O_{11}^2}{O_{1j} + p_jB} \approx 0 \quad \text{for all } j = 1, 2, \ldots, n. \]

As a consequence, the table looks more like

| \( (O_{11} - E_{11})^2 \) | \( (O_{12} - E_{12})^2 \) | \cdots | \( (O_{1n} - E_{1n})^2 \) |
| \( E_{11} \) | \( E_{12} \) | \cdots | \( E_{1n} \) |

and
\[ tv = \frac{(O_{11} - E_{11})^2}{E_{11}} + \frac{(O_{12} - E_{12})^2}{E_{12}} + \cdots + \frac{(O_{1n} - E_{1n})^2}{E_{1n}}. \]
Since these correspond to values obtained from the actual observed and expected frequencies (i.e., only those numbers used in the goodness of fit test as done by hand) we have
\[ tv = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}. \]

from the table

<table>
<thead>
<tr>
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<th>Category 1</th>
<th>Category 2</th>
<th>\cdots</th>
<th>Category n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed frequencies</td>
<td>( O_1 )</td>
<td>( O_2 )</td>
<td>\cdots</td>
<td>( O_n )</td>
</tr>
<tr>
<td>Expected frequencies</td>
<td>( E_1 )</td>
<td>( E_2 )</td>
<td>\cdots</td>
<td>( E_n )</td>
</tr>
<tr>
<td>Variations</td>
<td>( \frac{(O_1 - E_1)^2}{E_1} )</td>
<td>( \frac{(O_2 - E_2)^2}{E_2} )</td>
<td>\cdots</td>
<td>( \frac{(O_n - E_n)^2}{E_n} )</td>
</tr>
</tbody>
</table>